

Autonomous Control for a Differential Thrust ROV

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Abstract—Smaller autonomous underwater vehicles that use differential thrust for surge and yaw motion control has the advantage of low cost and, at the same time, increased maneuverability in yaw direction. However, since such vehicles are underactuated vehicles, design of an autonomous control system that enables the vehicle to autonomously track a predefined trajectory is challenging.

In this paper, we presented such an autonomous control system and implemented it on a small underactuated ROV with the use of unscented Kalman filter for vehicle localization, a underwater acoustic positioning system as the position sensor and a compass as the direction sensor. In designing the control law, the integrator backstep technique is used to achieve Lyapunov stability. Computer simulation and field tests have shown that the autonomous control system works well for the vehicle to track a predefined trajectory and the tracking error converged to a certain small value.

I. INTRODUCTION

Remotely Operated Vehicles (ROVs) and Autonomous Underwater Vehicles (AUVs) have been applied in a wide variety of areas. Recently, there has been a trend to use smaller autonomous underwater vehicles, both tethered and untethered. This research concerns underwater vehicles that use differential thrust for surge and yaw motion, with the advantage of increased maneuverability in the yaw direction. Unfortunately, such vehicles have limited control for lateral motion. Hence, when moving in a horizontal plane, they fall into the underactuated vehicle category because the dimension of the control vector is less than the degrees of freedom.

In this paper, we address the ability for an AUV to track a predefined trajectory, which might be parameterized with time. There has been a great deal of research on trajectory tracking problem for land vehicles, using various positioning systems, such as GPS, for navigation. Although much success has been achieved for the land vehicles trajectory tracking, it is still a problem for underwater vehicles. In the underwater world, vehicle positioning is much more difficult because several issues must be addressed including positioning accuracy, bandwidth, and possible time delay for underwater communication.

The main purpose for this work is to develop a control system for an AUV to track a predefined trajectory using an underwater acoustic positioning system. With the ability to track trajectories, the AUV will be able to carry out missions on its own without human intervention.

The vehicle used in this research is a VideoRay Pro III micro ROV (see Figure 1). It is a system designed for intensive, underwater operations. It has an open architecture that accommodates a wide variety of tools and sensors. The VideoRay Pro III system consists of a control console, a submersible robot and a tether deploying mechanism. The control console has a video display, joystick controls for horizontal and vertical movement, and a computer control interface. The submersible robot has two thrusters for horizontal movement control and one thruster for its vertical motion control, a pressure sensor for measuring depth, and a compass for measuring orientation. It also has an accessory connector allowing for field integration of various instruments and sensors.

A underwater acoustic positioning system was used for tracking the Videoray Pro III. The positioning system uses short base line (SBL) technology with three cabled sonar transducers. The best tracking performance of this acoustic positioning system is a nominal of ± 0.15 m RMS. The accuracy of the target position depends on the distance between the surface station transducers and the distance between the target and the transducers.

To allow the robot to track a predefined trajectory, an autonomous control system was developed. The block diagram of the trajectory tracking system used for the VideoRay Pro III is shown in Figure 2. It primarily consists of a trajectory reference input which could be generated by some high level mission planning algorithm, a control system that outputs desired control parameters such as surge force and yaw torque to drive the vehicle to follow the trajectory, and a physical system consisting of the vehicle itself, the sensor onboard the vehicle and the underwater acoustic positioning system. The control system is composed of a planar trajectory tracking controller, a depth controller, a bearing controller, a command conversion and an unscented Kalman filter (UKF). The controller and the unscented Kalman filter are described in the following sections. Note that for the estimator/predictor to generate accurate state estimation, an accurate dynamic model of the VideoRay Pro III is required.

The working principle of this entire trajectory tracking system is described as follows:

- 1) The control system takes the reference trajectory and decides in which mode it will run. The reference trajectory contains the horizontal position information x, y and their first three time derivatives: $\dot{x}, \dot{y}, \ddot{x}, \ddot{y}, \dddot{x}, \dddot{y}$.



Fig. 1. VideoRay Pro III

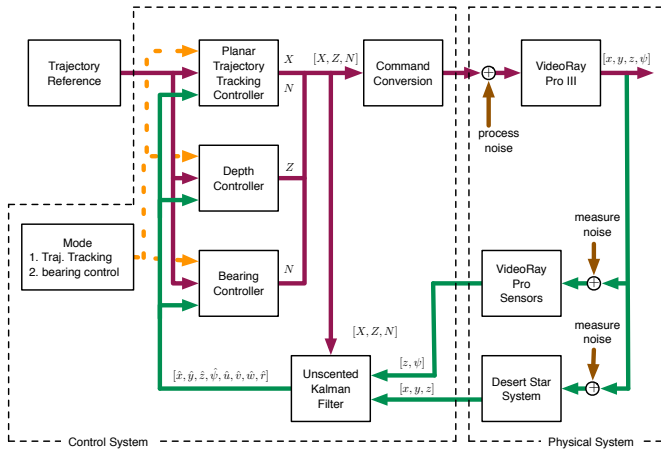


Fig. 2. Architecture of Entire Tracking System for VideoRay Pro III

- 2) The control system then computes the control input based on the reference trajectory and the state feedback from the unscented Kalman filter;
- 3) The control system converts the control input of surge force X , yaw torque N and heave force Z into the thruster control command and sends them to the VideoRay Pro III.
- 4) Meanwhile, the unscented Kalman filter predicts the system states $[x \ y \ z \ \psi \ u \ v \ w \ r]$ based upon the control input. Whenever there are measurements from the physical system coming in, the UKF corrects the system states with the knowledge of the measurement models.

The controller works at a sampling frequency of 25 Hz. The sensor system on board VideoRay Pro III provides the information of heading angle ψ and depth z , also at the bandwidth of 25 Hz. The acoustic positioning system works at a lower bandwidth of about one measurement every 1.5 to 3 seconds, depending on the distance between the sonar transducers and the vehicle being tracked.

The performance of the trajectory tracking system will be investigated by using simulations and experiments in the following sections.

II. BUILDING THE VEHICLE'S HYDRODYNAMIC MODEL

An accurate dynamic model is crucial to the realization of autonomous control for an underwater vehicle [10], [11]. However, the modeling and control of underwater vehicles is difficult. The governing dynamics of underwater vehicles are fairly well understood, but they are difficult to handle for practical design and control purposes [8], [3]. The problem includes many nonlinearities and modeling uncertainties. These hydrodynamic and inertial nonlinearities are present due to coupling between the degrees of freedom [4]. The presence of these non-linear dynamics requires the use of a numerical technique to determine the vehicle response to thruster inputs and external disturbances over the wide range of operating conditions.

In general, modeling techniques tend to fall into two categories [5]:

- 1) Predictive methods based on either Computational Fluid Dynamics or strip theory, and
- 2) Experimental techniques.

The predictive methods calculate the vehicle's dynamic motion parameters from the vehicle's design. It has the advantages of low cost, being easy to implement, and being able to carry out even before the vehicle has been built, but it has the disadvantage of less accuracy.

In contrast, experimental techniques are usually carried out by using towing tank testing and more recently by system identification methods, and have the advantage of being more accurate. They are usually more costly as well.

These two techniques have been used to build the hydrodynamic model for the VideoRay Pro III ROV. The ROV is considered as a 6 degrees of freedom (DOF) free body in space, namely *surge*, *sway*, *heave*, *pitch*, *roll* and *yaw* motions. The control of the vehicle is only available in the surge, heave, and yaw motion. Equal and differential thrust from the horizontal thrusters provide control in surge motion and yaw motion respectively.

The mathematical model of an underwater vehicle can be expressed, with respect to a local body-fixed reference frame, by the nonlinear equations of motion in matrix form [4]:

$$\mathbf{M}\dot{\nu} + \mathbf{C}(\nu)\nu + \mathbf{D}(\nu)\nu + \mathbf{g}(\eta) = \tau \quad (1)$$

$$\dot{\eta} = \mathbf{J}(\eta)\nu \quad (2)$$

where:

$\mathbf{M} = \mathbf{M}_{\text{RB}} + \mathbf{M}_{\text{A}}$, (sum of mass matrices for rigid body and added mass);

$\mathbf{C}(\nu) = \mathbf{C}_{\text{RB}}(\nu) + \mathbf{C}_{\text{A}}(\nu)$, (sum of Coriolis and Centripetal matrices for rigid body and added mass);

$\mathbf{D}(\nu) = \mathbf{D}_{\text{quad.}}(\nu) + \mathbf{D}_{\text{lin.}}(\nu)$, (damping matrices for rigid body and added mass);

$\mathbf{g}(\eta)$ is the hydrostatic restoring force matrix,

τ is the thruster input vector;

$\mathbf{J}(\eta)$ is the coordinate transform matrix which brings the inertial frame into alignment with the body-fixed frame.

The entries in the M_A , $C(\nu)$ and $D(\nu)$ matrices can be expressed using hydrodynamic derivatives. Theoretically, the hydrodynamic derivatives can be determined using an approach called strip theory [9]. However, the derivatives produced using this approach are often inaccurate and sometimes unsatisfactory. Validation of these derivatives is always desired.

We have used the strip theory and experimental method as well, to obtain the VideoRay's hydrodynamic model. And we found out that the derivatives estimated using strip theory are in good agreement with those later obtained by experiment.

Since, in this model, the movements in all the six degrees are fully coupled with each other, therefore, it is very difficult to determine the hydrodynamic derivatives using experimental method. We used a simplified decoupled vehicle model, based on the fact that

- 1) the weight and buoyancy distribution of the VideoRay Pro III will always force the vehicle to return back to the zero pitch and zero roll state. Therefore, we assume for all time $\phi = 0$, $\theta = 0$, $p = 0$ and $q = 0$,
- 2) the thrusters of the VideoRay Pro III only have effect in surge, heave and yaw motion.

The system can be broken down into two non-interacting subsystems:

- 1) x, y, ψ, u, v, r for horizontal plane motion
- 2) z, w for vertical plane motion

The decomposition also supports the idea that any control action for the surge direction is implemented using balanced thrusts from both side thrusters; and any control action for the yaw direction is implemented using differential thrust.

Assuming the vehicle is always in the zero-pitch and zero-roll state, *i.e.*, $\phi = 0$ and $\theta = 0$, we can write the decoupled models as follows.

- The model for horizontal plane motion:

$$m_{11}\dot{u} = -m_{22}vr + X_uu + X_{u|u}|u| + X, \quad (3)$$

$$m_{22}\dot{v} = m_{11}ur + Y_vv + Y_{v|v}|v|, \quad (4)$$

$$I\dot{r} = N_{r}r + N_{r|r}|r| + N, \quad (5)$$

where

- m_{11} = the (1,1) entry of the vehicle inertia matrix M ,
- m_{22} = the (2,2) entry of the vehicle inertia matrix M ,
- I = vehicle's moment of inertia about the z axis, which is the (6,6) entry of the vehicle inertia matrix M ,
- $X_u, X_{u|u}$ = linear and quadratic hydrodynamic coefficients in the surge direction,
- $Y_v, Y_{v|v}$ = linear and quadratic hydrodynamic coefficients in the sway direction,
- X = external force acting on the vehicle in the surge direction,
- N = external torque acting on the vehicle about the z axis.

- The model for vertical plane motion:

$$m_{33}\dot{w} = Z_w w + Z_{w|w}|w| + Z \quad (6)$$

where

m_{33} = the (3,3) entry of the vehicle inertia matrix M ,

$Z_w, Z_{w|w}$ = linear and quadratic hydrodynamic coefficients in the heave direction,

Z = external force acting on the vehicle in the heave direction,

This decoupled model will facilitate the design for the trajectory tracking controllers, which will be described in the following sections. The hydrodynamic derivatives for this model have been determined in the flume in the Fluid Lab in University of Waterloo. And the obtained model is verified in the surge and yaw modes[13].

III. DESIGNING THE ESTIMATOR/PREDICTOR

Having accurate VideoRay Pro III motion information, namely its position information x, y, z, ψ and velocity information u, v, w, r , is crucial for the trajectory tracking controller to work properly. Unfortunately, among these parameters only the 3-dimension position information (x, y, z) and heading information (ψ) are available from the vehicle's sensor system and underwater acoustic positioning system; the velocity could not be measured directly. Also, the position information obtained through the measurement is uncertain due to noise and other imperfections. To handle this problem, estimation (or filtering) is applied to the measurements.

The extended Kalman filter (EKF) is a popular choice for estimating the system state. Unfortunately, it has two significant flaws in this application. First, it requires the derivation of the Jacobian matrices, *i.e.*, the linear approximation to the nonlinear functions, which can be complex and causes implementation difficulties. Second, these linearizations can lead to filter instability if the timestep intervals are not sufficiently small [7]. Besides these flaws, the EKF is not suitable for discontinuous process models [6], where the representation of the nonlinear functions and probability distribution of interest is not adequate.

Julier and Uhlman [7] proposed the unscented Kalman filter (UKF) which solved the flaws with the EKF by using a deterministic sampling approach. In UKF the system state distribution is approximated by a Gaussian random variable represented using a minimal set of carefully chosen sample points, which are called sigma points. The sigma points completely capture the true mean and covariance of the system state, and when propagated through the non-linear system, captures the posterior mean and covariance accurately to the second order Taylor series expansion.

To use the UKF, the state random variable is redefined as the concatenation of the original state variables \mathbf{x}_t and noise variables \mathbf{m}_t and \mathbf{n}_t , which is called the augmented state

variable:

$$\mathbf{x}_t^a = \begin{bmatrix} \mathbf{x}_t \\ \mathbf{m}_t \\ \mathbf{n}_t \end{bmatrix} \quad (7)$$

The covariances associated with \mathbf{x}_t , \mathbf{m}_t and \mathbf{n}_t are \mathbf{P}_t , \mathbf{Q} , and \mathbf{R} , respectively. Here, we assume that the \mathbf{Q} and \mathbf{R} are constant. The UKF algorithm will use the augmented state variable to predict the system state with control input, and then correct the predicted states using the measurements. The complete UKF algorithm implementation for our control system can be found in [12].

IV. DESIGNING THE TRAJECTORY TRACKING CONTROLLER

In this section we will develop a control law to allow the VideoRay Pro III to track a horizontal planar position trajectory $\Phi(t)$ defined by

$$\Phi(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

The key step in the approach is to apply the backstepping technique to track only two position variables instead of the entire three dimensional configuration $[x, y, \psi]$ in the horizontal plane, based on the work by Aguiar *et al.* in [1]. However, Aguiar *et al.* did not consider in their vehicle's model the quadratic damping terms, which are significant in our application. We developed the trajectory tracking controller with the quadratic drag terms considered in the model. The resulting control law can be proven globally Lyapunov stable with the condition that the actuator never saturates.

We will start by describing the kinematic and dynamic equations for the VideoRay Pro III, followed by the formulation of the corresponding problem of planar trajectory tracking control. Finally, we will derive the solution to this problem by utilizing an integrator backstepping technique.

A. Vehicle Modeling

From previous sections, the general kinematic equations and dynamic equations of motion of the vehicle can be developed using an earth-fixed coordinate frame $\{U\}$ and a body-fixed coordinate frame $\{B\}$ that are depicted in Figure 3.

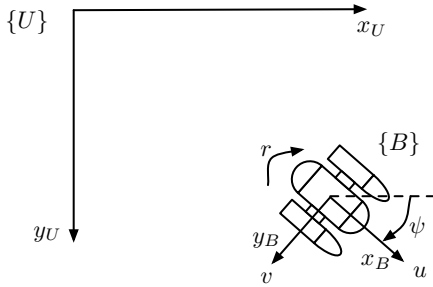


Fig. 3. Body-fixed $\{B\}$ and earth-fixed $\{U\}$ coordinate frames

In the horizontal plane, the kinematic equations of motion for the vehicle can be reduced in matrix format as:

$$\dot{p} = R(\psi)\nu, \quad (8)$$

$$\dot{\psi} = r, \quad (9)$$

$$M\dot{\nu} = -S(r)M\nu + D_\nu(\nu)\nu + gX, \quad (10)$$

$$J\dot{r} = d_r(r)r + N. \quad (11)$$

where

$p = \begin{bmatrix} x \\ y \end{bmatrix}$ is the position vector, expressed in the $\{U\}$ frame;

$\nu = \begin{bmatrix} u \\ v \end{bmatrix}$ is the velocity vector, expressed in the $\{B\}$ frame;

$R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix}$ is the rotation matrix;

$S(r) = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$ is a skew-symmetric matrix;

$M = \begin{bmatrix} m_{11} & 0 \\ 0 & m_{22} \end{bmatrix}$ is the mass matrix;

$D_\nu(\nu) = \begin{bmatrix} X_u + X_{u|u}|u| & 0 \\ 0 & Y_v + Y_{v|v}|v| \end{bmatrix}$ is the damping coefficient matrix, which is negative definite and time varying;

$d_r(r) = N_r + N_{r|r}|r|$, which is negative definite and time varying;

$$g = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

B. Control Laws

Given the above kinematic and dynamic equations of motion, we would like to design a controller such when tracking a sufficiently smooth time-varying desired trajectory $p_d = [x_d \ y_d]^T$, all the closed-loop signals are globally bounded and the tracking error $\|p - p_d\|$ converges exponentially to a neighborhood of the origin that can be made arbitrarily small.

We use the integrator backstepping method to derive the control law for surge force X and yaw torque N to track trajectories in horizontal motion plane[12]:

$$X = X(p, p_d, \dot{p}_d, \ddot{p}_d, \delta), \quad (12)$$

$$N = N(p, \dot{p}, p_d, \dot{p}_d, \ddot{p}_d, \ddot{\ddot{p}}_d) \quad (13)$$

where:

$\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$ is a controller constant vector that can be made arbitrarily small.

Note that in the above control law, the surge force X is a function of the vehicles current position p , desired position p_d and the first two derivatives of the desired position at certain time point. The yaw torque even needs the first four derivatives of the desired position. The parameters δ_1 and δ_2 can be used to adjust the controller accuracy. Although theoretically they can be made arbitrarily small, it is usually set to a value that will not cause the controller to be overly sensitive to system noise.

It can be proved, in [2] that given a three-times continuously differentiable time-varying desired trajectory $p_d : [0, \infty) \rightarrow \mathbf{R}^2$ with its first three derivatives bounded, consider the closed-loop system Σ consisting of the underactuated vehicle model and feedback controller,

- 1) for any initial condition the solution to Σ exists globally, all closed-loop signals are bounded, and the tracking error $\|p(t) - p_d(t)\|$ satisfies

$$\|p(t) - p_d(t)\| \leq e^{-\lambda t} c_0 + \varepsilon, \quad (14)$$

where $\lambda, c_0, \varepsilon$ are positive constants. From these, only c_0 depends on initial conditions.

- 2) By appropriate choice of the controller parameters k_e, K_φ, k_{z2} , any desired values for ε and λ in Equation 14 can be obtained.

V. COMPUTER SIMULATION AND FIELD TESTS

A. Simulation Results

The controller performance was studied by computer simulation. In this simulation, the vehicle is initially located at the origin $(0, 0)$ with heading angle $\psi = 0$. The reference trajectory is a circle with the following parameters: radius=2.5 m, motion speed is 0.3 m/s, depth=1 m, starting point $(2.5, 0)$ at time $t_0 = 0$ and stopping time at $t_f = 45$.

A process noise for the vehicle that has the covariance of $\mathbf{Q} = \text{diag}(0.05, 0.05, 0.05)$ is added to its three thrusters. The measurement variances for heading angle and depth are obtained by tests, which are 0.05 rad^2 and 0.03 m^2 respectively. The variance for position measurement from the acoustic positioning system is given by the manufacturer as 0.4 m^2 , which is also confirmed by test. The heading and depth measurement will be obtained at the frequency of 25 Hz, while the position information from the underwater acoustic positioning system will come out once every 2 seconds.

The simulation result is shown in Figure 4. The desired trajectory in x - y plane is shown with dashed blue line. The position measurements are shown with asterisks. The red dash-dot line shows the simulated vehicle trajectory that is calculated with the 6-DOF nonlinear dynamic model of the VideoRay Pro III developed in the previous section. The green solid line shows the estimated vehicle position calculated by the UKF based on the decoupled dynamic model in Equation 3, 4 and 5.

From the simulation, we see that the proposed controller makes the tracking error converge to a very small value. Therefore, the performance of the tracking system is considered satisfactory in simulation.

B. Test Results

Seven tests were conducted with the desired trajectory being straightline starting from a position of $(3 \text{ m}, 0)$ in the x - y plane and moving to the target position of $(3 \text{ m}, 7.8 \text{ m})$; Once the vehicle arrives the target position, it returns back and moves

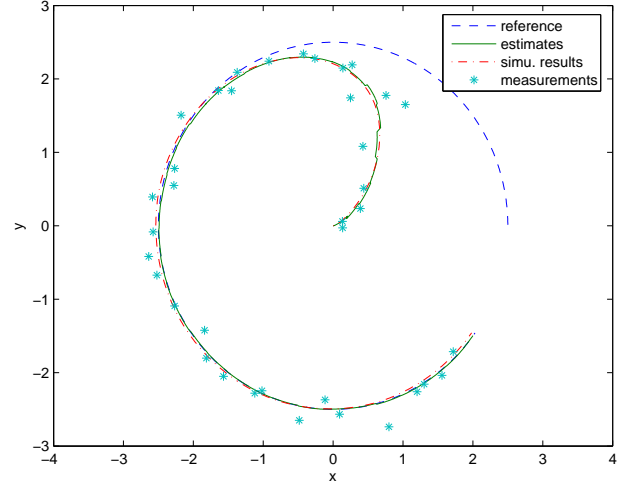


Fig. 4. Planar trajectory tracking simulation

to the original starting points. During the entire trajectory, the desired depth is set to 0.5 m below water surface.

The statistics of the test results are listed in Table I. The results showed that the controller works well in tracking the trajectory. The mean error of the actual trajectory is within 0.2 m. The maximum variance is 0.245 m.

experiment #	mean value of x	variance	standard deviation
1	3.19	0.185	0.43
2	2.91	0.245	0.50
3	3.10	0.017	0.13
4	3.04	0.019	0.14
5	3.19	0.032	0.18
6	2.89	0.070	0.27
7	2.97	0.024	0.15

TABLE I
STRAIGHT LINE TEST RESULTS

The reference and actual trajectory tracked for one of the tests are shown in Figure 5. The VideoRay Pro III was launched at $(2 \text{ m}, 0)$, which is 1 meters off the desired starting point. Note that the vehicle moves towards the reference trajectory and eventually converges to a neighborhood of the trajectory. The controller parameters were first obtained by simulation and then fine tuned by trial and error in the experiments.

Note that the acoustic positioning system returns the position information with relatively large noise (with standard deviation of about 0.15 m). Also, not only the position measurements are noisy, they are coming up at vary time intervals as well. These noisy measurements are filtered out through the unscented Kalman filter. In the plots of position variance shown in Figure 6 and Figure 8, we can see that the variances grow when there are no measurements coming in. At the moment the measurement is obtained, the position covariance is reduced as the result of the sensor fusion achieved through the UKF.

Figure 7 shows the yaw angle result. The tests show that the performance of the trajectory tracking controller relies heavily on the accuracy and performance of the compass. However, the compass is affected in a large extent by the environment, especially for the indoor test. As we can see, when the VideoRay Pro III is heading forward, the compass response is relatively smooth. When the vehicle is moving backwards returning to the starting point, the response of the compass is very sensitive to the environment and exhibits a large error.

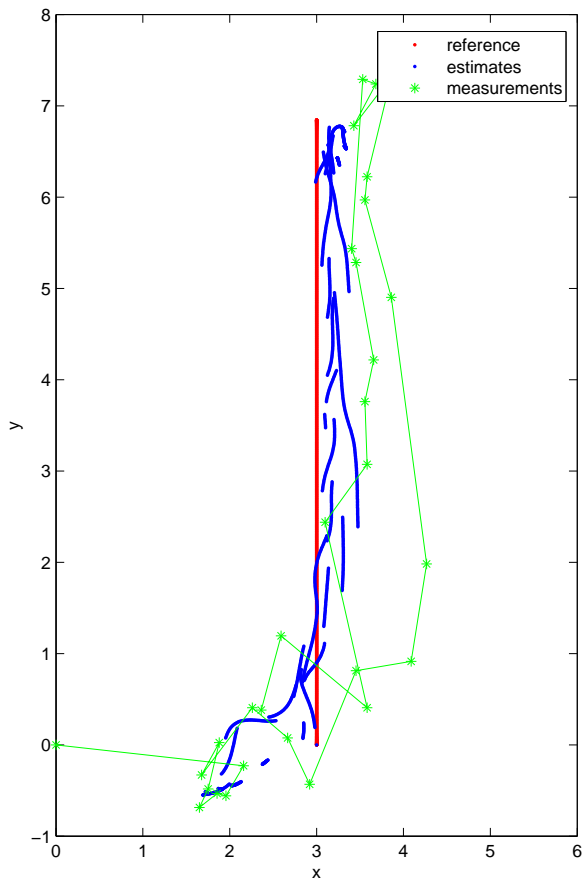


Fig. 5. Straight line test trajectory: The reference trajectory is a straightline starting from (3, 0) to (3, 7.8) and then returning back along the same line to the starting point; The green asterisks show the acoustic positioning measurements as the vehicle is moving along the trajectory. The blue line segments show the estimates of the vehicle's position achieved through the UKF.

VI. CONCLUSION

In this work, we presented the development and analysis of a dynamical vehicle model as well as a trajectory tracking controller for the VideoRay Pro III ROV.

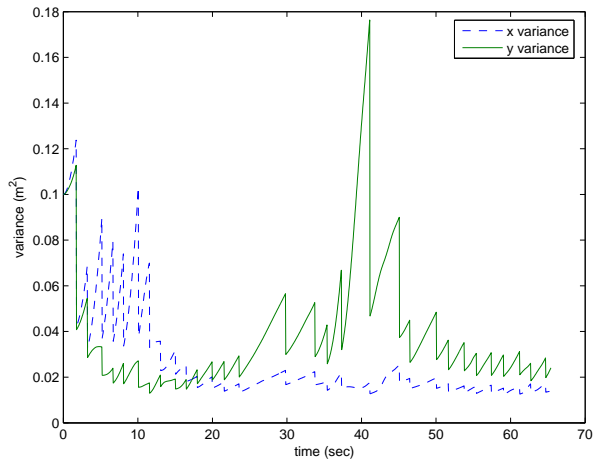


Fig. 6. Straight line test: x and y variance

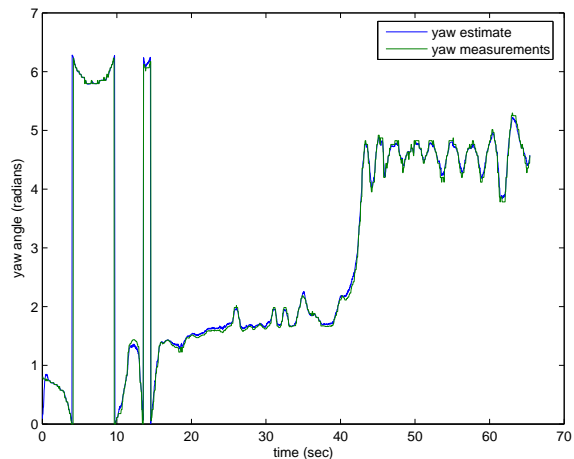


Fig. 7. Test results for the yaw angle

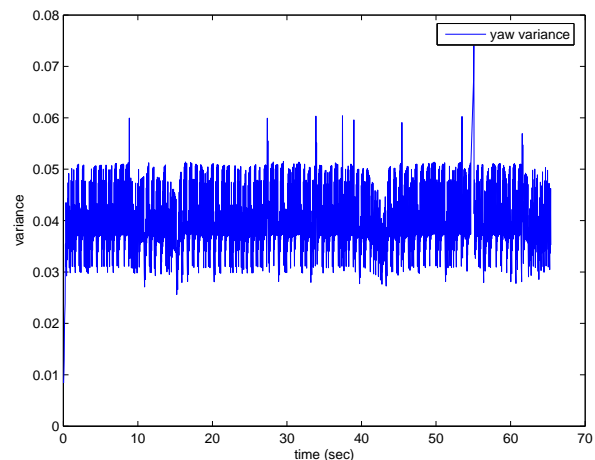


Fig. 8. Straight line test: z variance

With respect to vehicle modeling, our basic approach was to model the vehicle as having constant inertial and added mass characteristics, decoupled vehicle motion due to its symmetric geometric profile, and low operating speed. The hydrodynamic coefficients of the model were determined by both theoretical and experimental approaches. The model was verified by experiments and exhibited adequate accuracy for the design of trajectory tracking controller.

A state estimator was designed using the unscented Kalman filter. Since the vehicle's model exhibits high non-linearity and the vehicle has a large operating range, the unscented Kalman filter was used to overcome several drawbacks that come with the traditional extended Kalman filter, which has been widely used for state estimation.

With the decoupled vehicle model developed, we designed a trajectory tracking controller using the integrator backstepping technique, based on the work done by Aguiar *et al.* [1]. As a result, we obtained a Lyapunov stable trajectory tracking controller, considering the quadratic damping terms in the dynamical model which were neglected by Aguiar *et al.* For the depth and heading control, we developed a sliding mode controller which provided robust tracking control despite the fact that the vehicle model may have inaccurate parameters and may be subject to unmodeled disturbances during its mission.

The controllers were validated by simulation and experiments. For validation, we used the acoustic underwater positioning system as our position measuring device. In the simulation, the closed-loop system was shown to be stable to track a feasible predefined trajectory. In the experiments, we have shown that the controller was working well in tracking a straight line trajectory. The tracking error was within a certain range that is acceptable to the AUV of its category.

Currently, the controller developed heavily relies on the performance the onboard compass to obtain bearing information. Unfortunately, the onboard compass exhibits high nonlinearity measuring heading and is extremely affected by its working environment. This made it difficult to control the vehicle in tracking the trajectory. The compass inaccuracy could be compensated with a inertial measurement unit (IMU) which will be our future work.

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