E190Q – Lecture 10
Autonomous Robot Navigation

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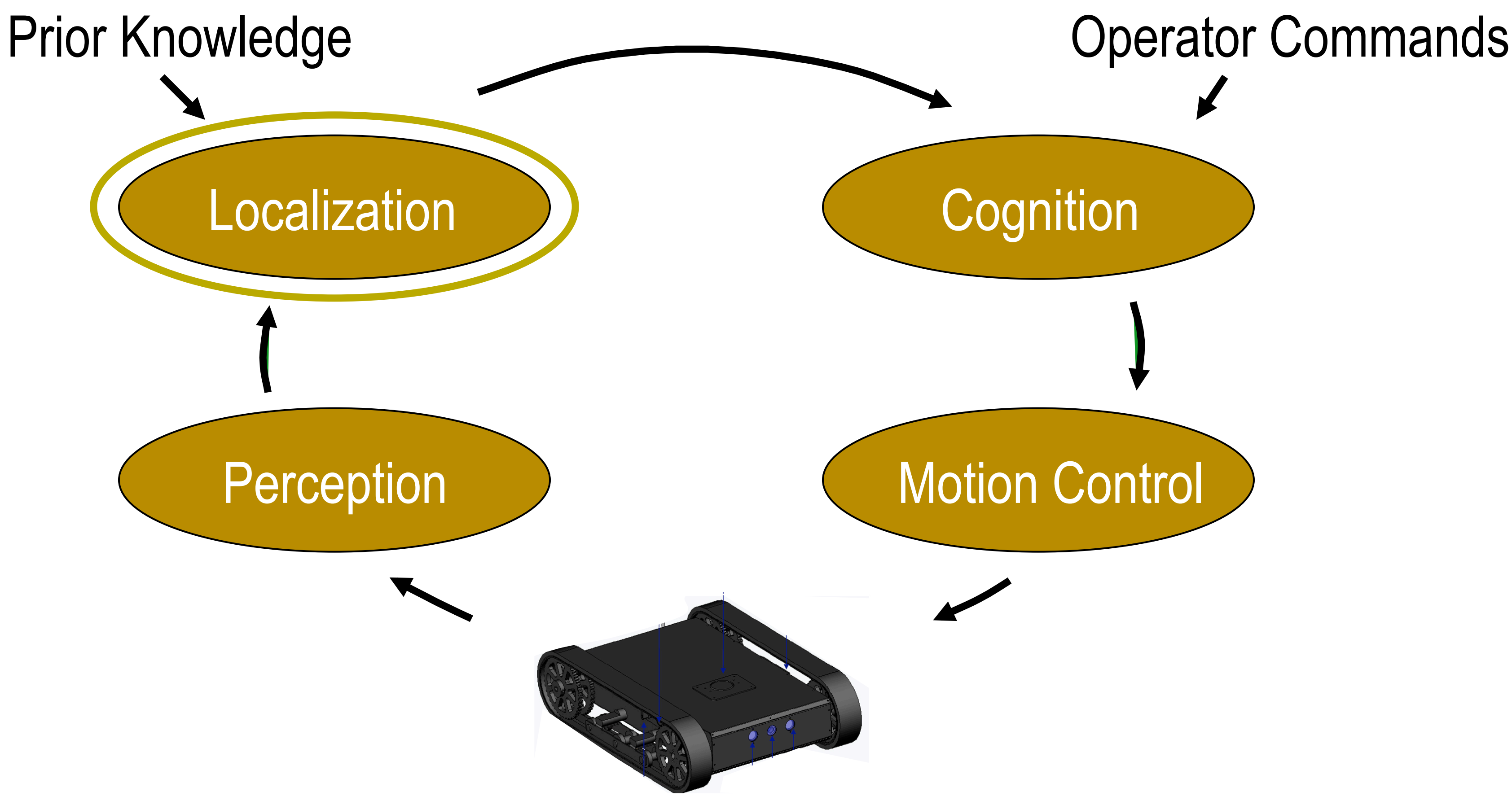
Semester: Spring 2015

Figures courtesy of Siegwart & Nourbakhsh

Kilobots

https://www.youtube.com/watch?v=2IAluwgAFD0

Control Structures
Planning Based Control



Particle Filter Localization: Outline

1. Particle Filters
    1. What are particles?
    2. Algorithm Overview
    3. Algorithm Example
    4. Using the particles
2. PFL Application Example

What is a particle?

- Like Markov localization, PFs represent the belief state with a set of **discrete** possible states, and assigning a **probability** of being in each of the possible states.

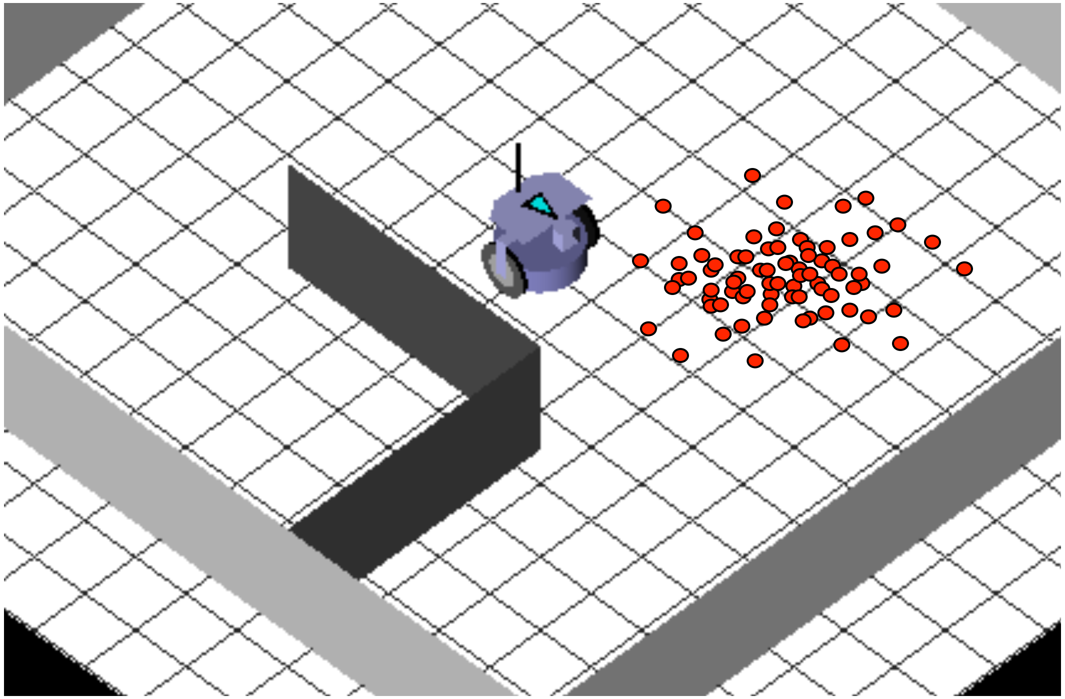
- Unlike Markov localization, the set of possible states are not constructed by discretizing the configuration space, they are a **randomly** generated set of "**particles**".

What is a particle?

- A particle is an individual state estimate.
- A particle is defined by its:
  1. State values that determine its location in the configuration space, e.g. $\mathbf{x} = [\, x \; y \; \theta \,]$
  2. A probability that indicates it's likelihood.

What is a particle?

- Particle filters use many particles to for representing the belief state.



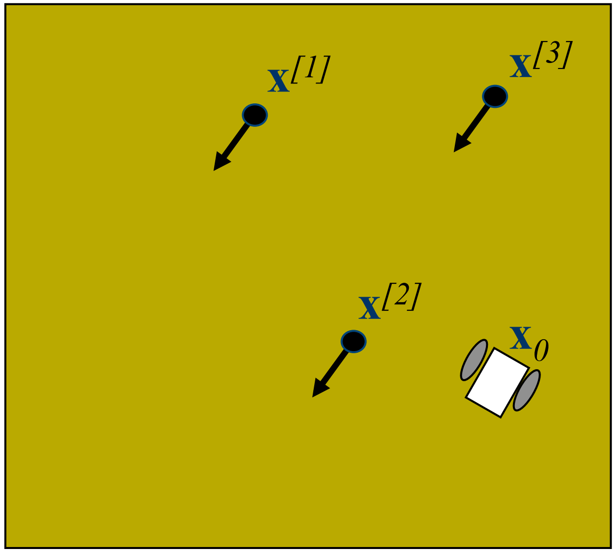
What is a particle?

- Example:
  - A Particle filter uses *3* particles to represent the position of a (white) robot in a square room.
  - If the robot has a perfect compass, each particle is described as:

$$\mathbf{x}^{[1]} = [\, x^1 \; y^1 \,]$$

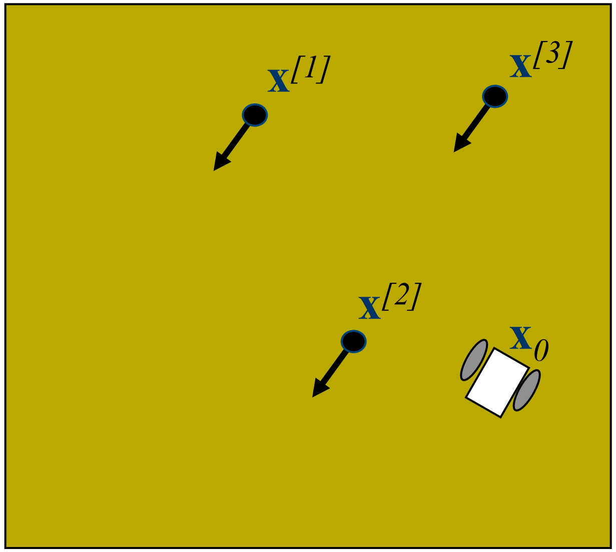
$$\mathbf{x}^{[2]} = [\, x^2 \; y^2 \,]$$

$$\mathbf{x}^{[3]} = [\, x^3 \; y^3 \,]$$



What is a particle?

- Example:
  - Each of the particles $\mathbf{x}^{[1]}$, $\mathbf{x}^{[2]}$, $\mathbf{x}^{[3]}$ also have associated weights $w^{[1]}$, $w^{[2]}$, $w^{[3]}$.
  - In the example below, $\mathbf{x}^{[2]}$ should have the highest weight if the filter is working.



What is a particle?

- The user can choose how many particles to use:
  - More particles ensures a higher likelihood of converging to the correct belief state
  - Fewer particles may be necessary to ensure real-time implementation

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Markov Localization Particle Filter

- Algorithm (Initialize at $t = 0$):
  - Randomly draw N states in the work space and add them to the set $\mathbf{X}_0$.

$$\mathbf{X}_0 = \{\mathbf{x}_0^{[1]}, \mathbf{x}_0^{[2]}, \ldots, \mathbf{x}_0^{[N]}\}$$

  - Iterate on these N states over time (see next slide).

Markov Localization
Particle Filter

- Algorithm (Loop over time step t):

1. For $i = 1 \ldots N$
2. Pick $\mathbf{x}_{t-1}^{[i]}$ from $\mathbf{X}_{t-1}$
3. Draw $\mathbf{x}_t^{[i]}$ with probability $P(\mathbf{x}_t^{[i]} \mid \mathbf{x}_{t-1}^{[i]}, o_t)$
4. Calculate $w_t^{[i]} = P(z_t \mid \mathbf{x}_t^{[i]})$
5. Add $\mathbf{x}_t^{[i]}$ to $\mathbf{X}_t^{Predict}$

Prediction

6. For $j = 1 \ldots N$
7. Draw $\mathbf{x}_t^{[j]}$ from $\mathbf{X}_t^{Predict}$ with probability $w_t^{[j]}$
8. Add $\mathbf{x}_t^{[j]}$ to $\mathbf{X}_t$

Correction

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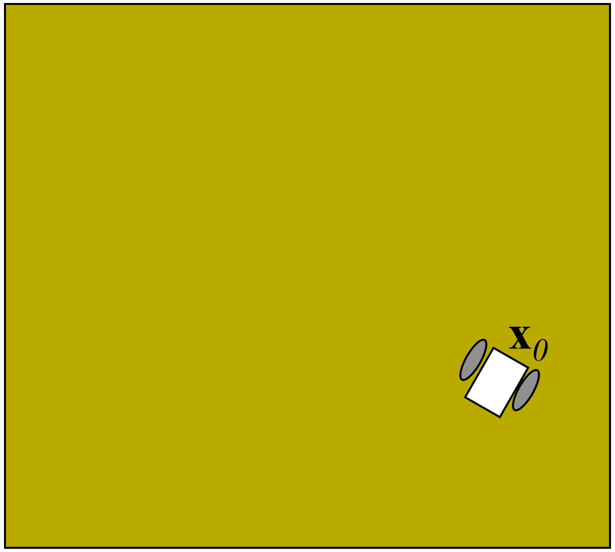
Particle Filter Example

- Provided is an example where a robot (depicted below), starts at some unknown location in the bounded workspace.

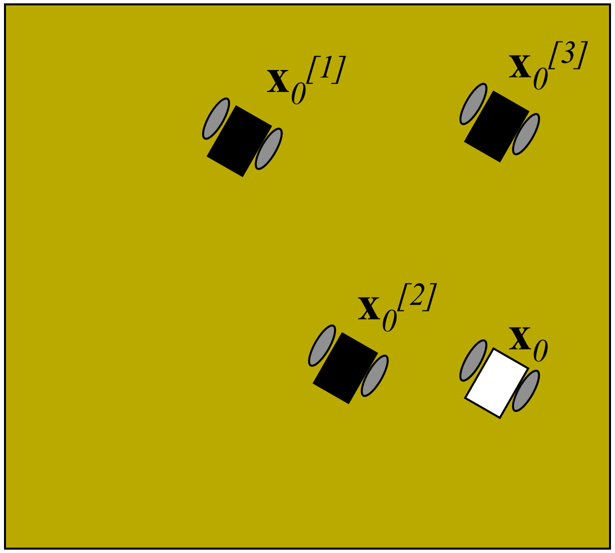


Particle Filter Example

- At time step t_0:
  - We randomly pick $N=3$ states represented as

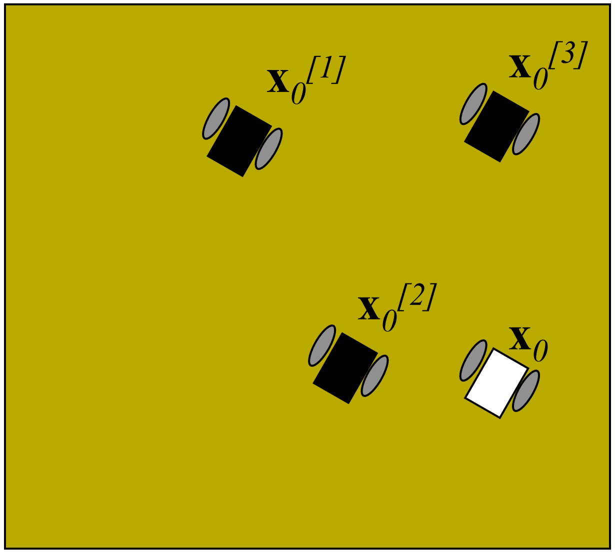
$$\mathbf{X}_0 = \{\mathbf{x}_0^{[1]}, \mathbf{x}_0^{[2]}, \mathbf{x}_0^{[3]}\}$$

  - For simplicity, assume known heading



Particle Filter Example

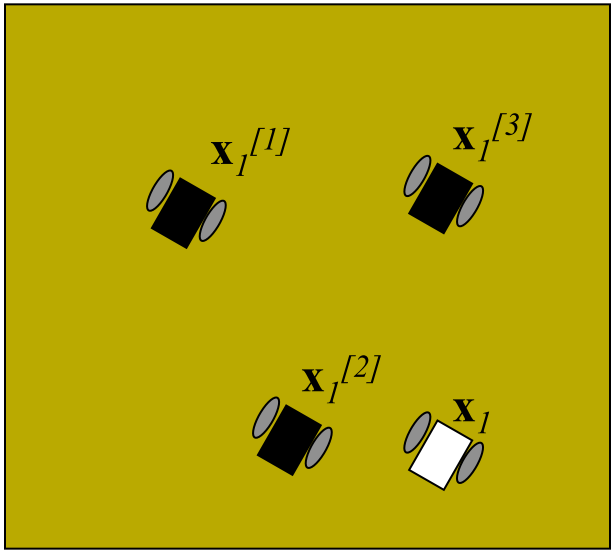
- The next few slides provide an example of one iteration of the algorithm, given $\mathbf{X}_0$.
  - This iteration is for time step t_1.
  - The inputs are the measurement z_1, odometry o_1



Particle Filter Example

- For Time step t_1:
  - Randomly generate new states by propagating previous states X_0 with o_1

$$\mathbf{X}_1^{Predict} = \{\mathbf{x}_1^{[1]}, \mathbf{x}_1^{[2]}, \mathbf{x}_1^{[3]}\}$$



Particle Filter Example

- For Time step t_1:
  - To get new states, use the motion model from lecture *3* to randomly generate new state $\mathbf{x}_1^{[i]}$.
  - Recall that given some Δs_r and Δs_l we can calculate the robot state in global coordinates:

$$\Delta x = \Delta s \cos(\theta + \Delta\theta / 2)$$

$$\Delta y = \Delta s \sin(\theta + \Delta\theta / 2)$$

$$\Delta\theta = \frac{\Delta s_r - \Delta s_l}{b}$$

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

Particle Filter Example

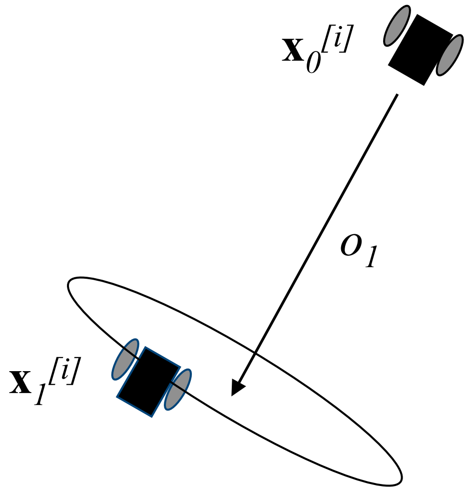
- For Time step t_1:
  - If you add some random errors ε_r and ε_l to Δs_r and Δs_l, you can generate a new random state that follows the probability distribution dictated by the motion model.
  - So, in the prediction step of the PF, the i^{th} particle can be randomly propagated forward using measured odometry $o_1 = [\ \Delta s_r\ \Delta s_l\ ]$ according to:

$$\Delta s_r^{[i]} = \Delta s_r + \text{rand}(\text{'norm'}, 0, \sigma_s)$$

$$\Delta s_l^{[i]} = \Delta s_l + \text{rand}(\text{'norm'}, 0, \sigma_s)$$

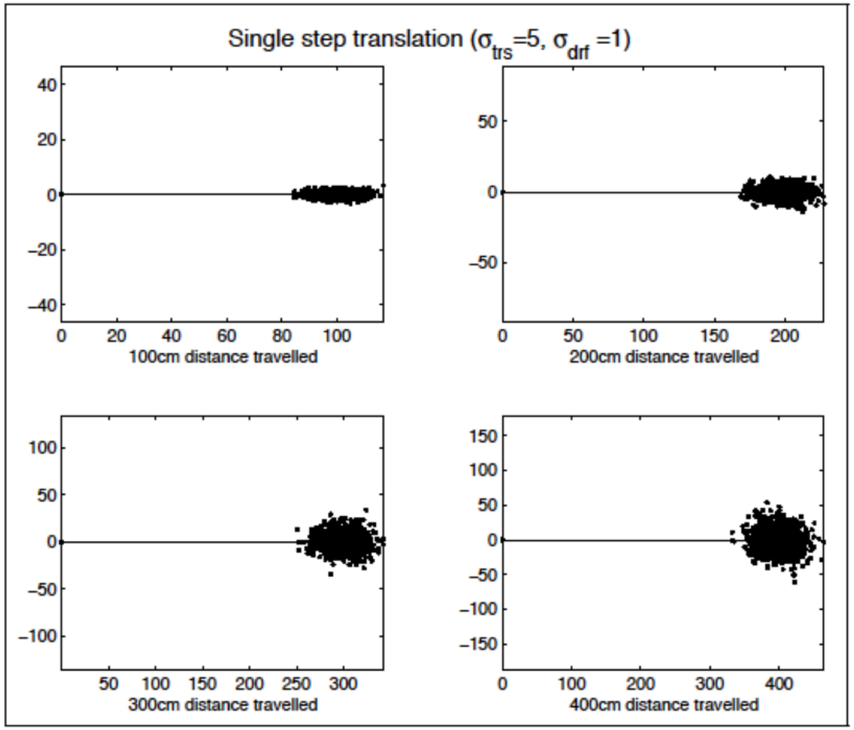
Particle Filter Example

- For Time step t_1:
  - For example:



Particle Filter Example

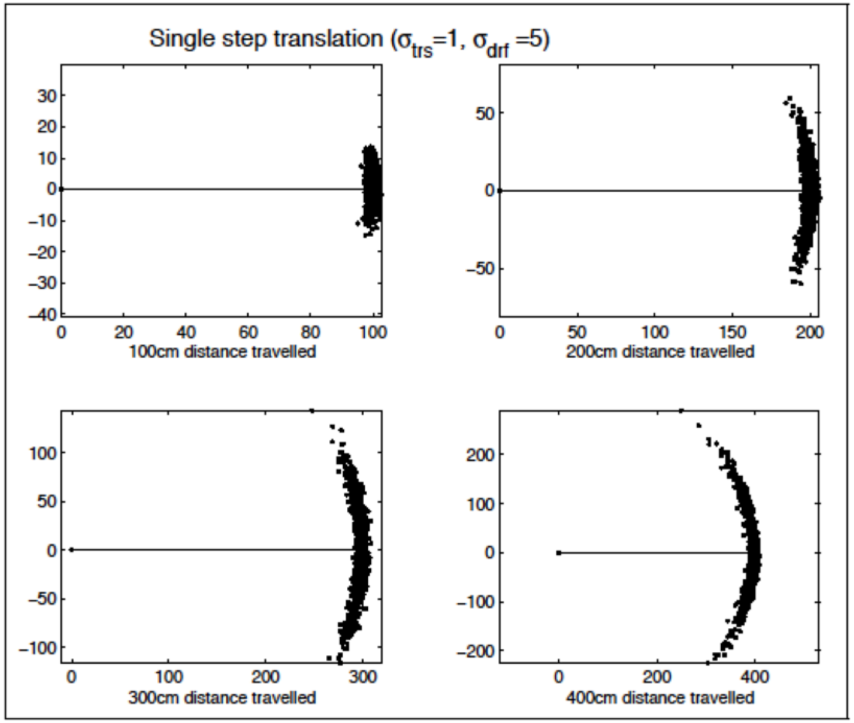
- Example Prediction Steps



Yiannis, McGill University, PF Tutorial

Particle Filter Example

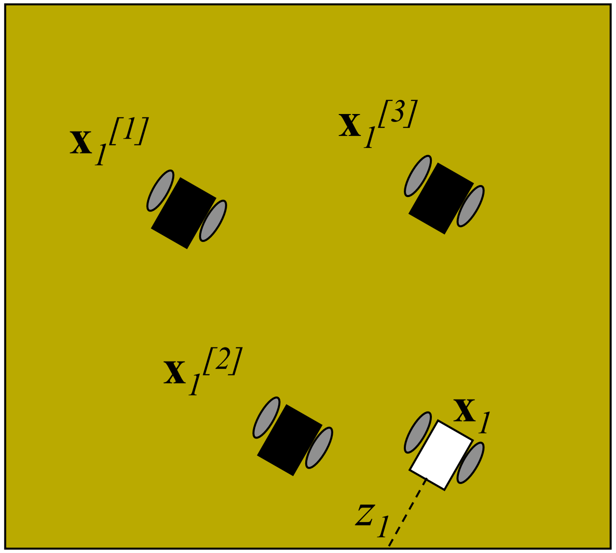
- Example Prediction Steps



Yiannis, McGill University, PF Tutorial

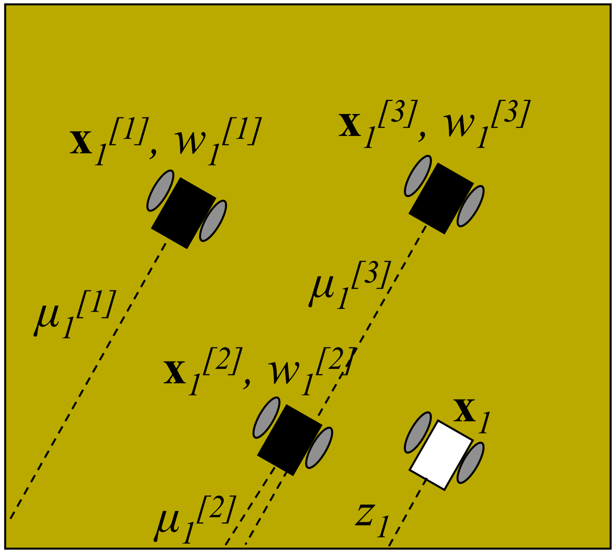
Particle Filter Example

- For Time step t_1:
  - We get a new measurement z_1, e.g. a forward facing range measurement.



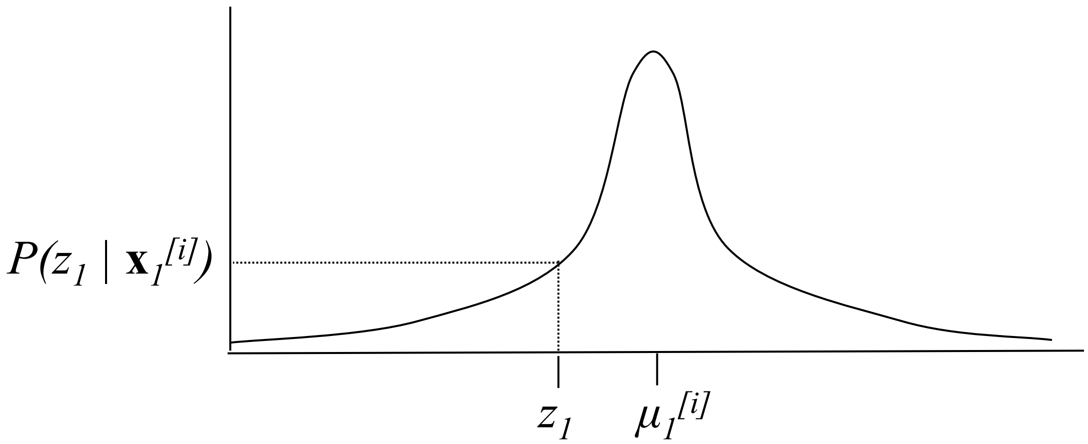
Particle Filter Example

- For Time step t_1:
  - Using the measurement z_1, and expected measurements $\mu_1^{[i]}$, calculate the weights $w^{[i]} = P(z_1 \mid \mathbf{x}_1^{[i]})$ for each state.



Particle Filter Example

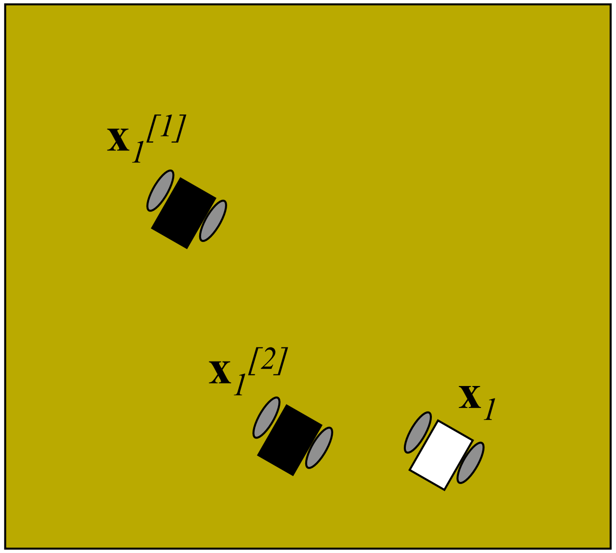
- For Time step t_1:
  - To calculate $P(z_1 \mid \mathbf{x}_1^{[i]})$ we use the sensor probability distribution of a single Gaussian of mean $\mu_1^{[i]}$ that is the expected range for the particle
  - The Gaussian variance is obtained from experiment.



Particle Filter Example

- For Time step t_1:
  - Resample from the temporary state distribution based on the weights $w_1^{[2]} > w_1^{[1]} > w_1^{[3]}$

$$\mathbf{X}_1 = \{\mathbf{x}_1^{[2]}, \mathbf{x}_1^{[2]}, \mathbf{x}_1^{[1]}\}$$



Particle Filter Example

- For Time step t_1:
  - How do we resample?
    - Exact Method
    - Approximate Method
    - Others…

Particle Filter Example

- An Exact Method

$w_{tot} = \sum_j w_j$

for $i=1..N$

$r = rand(\text{'uniform'}) * w_{tot}$

$j = 1$

$w_{sum} = w_1$

$while\ (w_{sum} < r)$

$j = j+1$

$w_{sum} = w_{sum} + w_j$

$\mathbf{x}_i = \mathbf{x}_j^{Predict}$

Particle Filter Example

- An Approximate Method

$w_{tot} = max_j \ w_j$
for $i = 1..N$

$w_i = w_i / w_{tot}$
if $w_i < 0.25$
add 1 copy of $\mathbf{x}_i^{Predict}$ to $\mathbf{X}^{TEMP}$
else if $w_i < 0.50$
add 2 copies of $\mathbf{x}_i^{Predict}$ to $\mathbf{X}^{TEMP}$
else if $w_i < 0.75$
add 3 copies of $\mathbf{x}_i^{Predict}$ to $\mathbf{X}^{TEMP}$
else if $w_i < 1.00$
add 4 copies of $\mathbf{x}_i^{Predict}$ to $\mathbf{X}^{TEMP}$

Particle Filter Example

- An Approximate Method (cont')

for $i = 1..N$

$$r = (int)\ rand(\text{'}uniform\text{'})*size(\mathbf{X}^{TEMP})$$

$$\mathbf{x}_i = \mathbf{x}_r^{TEMP}$$

Particle Filter Example

- NOTE:

  We should only resample when we get NEW measurements.

Particle Filter Example

- For Time step t_2:
  - Iterate on previous steps to update state belief at time step t_2 given $(\mathbf{X}_1, o_2, z_2)$.

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Additional Notes

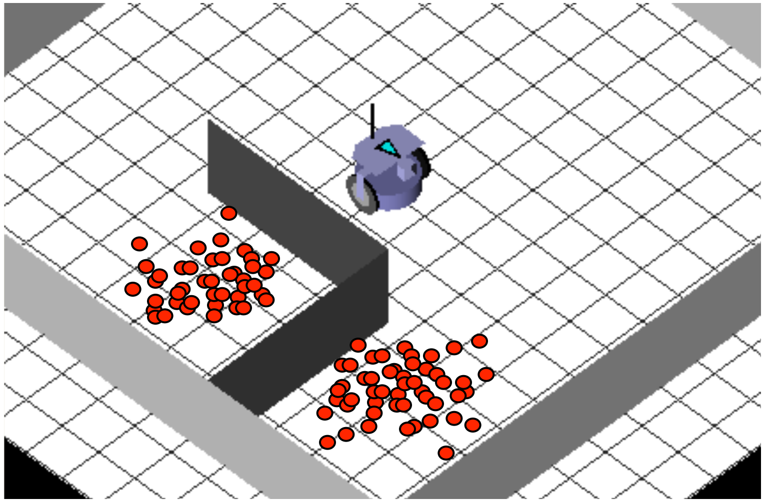
- How do we use the belief?
  - To control the robot, we often distill the belief into a lower dimension representation.
  - Examples:

$$\hat{\mathbf{x}}_1 = \frac{\sum_i w_1^{[i]} \mathbf{x}_1^{[i]}}{\sum_i w_1^{[i]}}$$

$$\hat{\mathbf{x}}_1 = \{ \mathbf{x}_1^{[i]} \mid w_1^{[i]} > w_1^{[j]} \ \forall\, j \neq i \}$$

Additional Notes

- How do we use the belief?
  - Sometimes we have several clusters
  - Lets introduce a new algorithm…



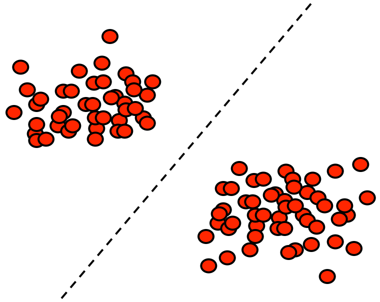
Additional Notes

- K-means Clustering
  - Given:

    A set of N data points $\mathbf{X} = \{ \mathbf{x}^{[1]}, \mathbf{x}^{[2]}, .. \mathbf{x}^{[N]} \}$

    The number of clusters $k \leq N$
  - Find:

    The k hyperplanes which best divide the data points into k clusters



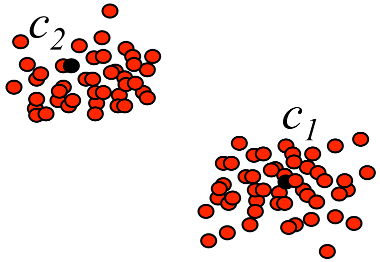
Additional Notes

- Subtractive Clustering
  - Given:

    A set of N data points $\mathbf{X} = \{ \mathbf{x}^{[1]}, \mathbf{x}^{[2]}, .. \mathbf{x}^{[N]} \}$

    Neighborhood Radius r_A
  - Find:

    The k data points which best divide the data points into k clusters



Additional Notes

- Subtractive Clustering Algorithm (initialization)

// Calculate Potential Values P_i

for $i = 1..N$

$$P_i = \sum_j exp(-||\mathbf{x}^{[i]} - \mathbf{x}_1^{[j]}||^2 / (0.5\, r_A)^2)$$

// Define first centroid center $\mathbf{c}_1$

$\mathbf{c}_1 = \{ \mathbf{x}_1^{[m]} \mid P_m > P_j \;\forall j \neq m \}$

$\text{PotVal}(\mathbf{c}_1) = P_m$

Chen, Qin, Jia Weighted Mean Subtractive Clustering Algorithm (2008)

Additional Notes

- Subtractive Clustering Algorithm (iterations)

$k = 1$

while ($!\ stoppingCriteria$)

// Update Potential Values

for $i = 1..N$

$$P_i = P_i - \mathrm{PotVal}(c_k)\ exp(\ -||\mathbf{x}^{[i]} - \mathbf{c}_k||^2 / (0.75\ r_A)^2)$$

// Calculate *k*^th centroid

$$\mathbf{c}_k = \{\ \mathbf{x}_1^{[m]}\ |\ P_m > P_j\ \forall\ j \neq m\ \}$$

$\mathrm{PotVal}(\mathbf{c}_k) = P_m$

$k=k+1$

Additional Notes

- Subtractive Clustering Algorithm (iterations)
  - The *stoppingCriteria* can take on many forms:

$$max_i(\mathrm{P}_i) < threshold$$

Additional Notes

- Subtractive Clustering Algorithm Example for $N = 7$

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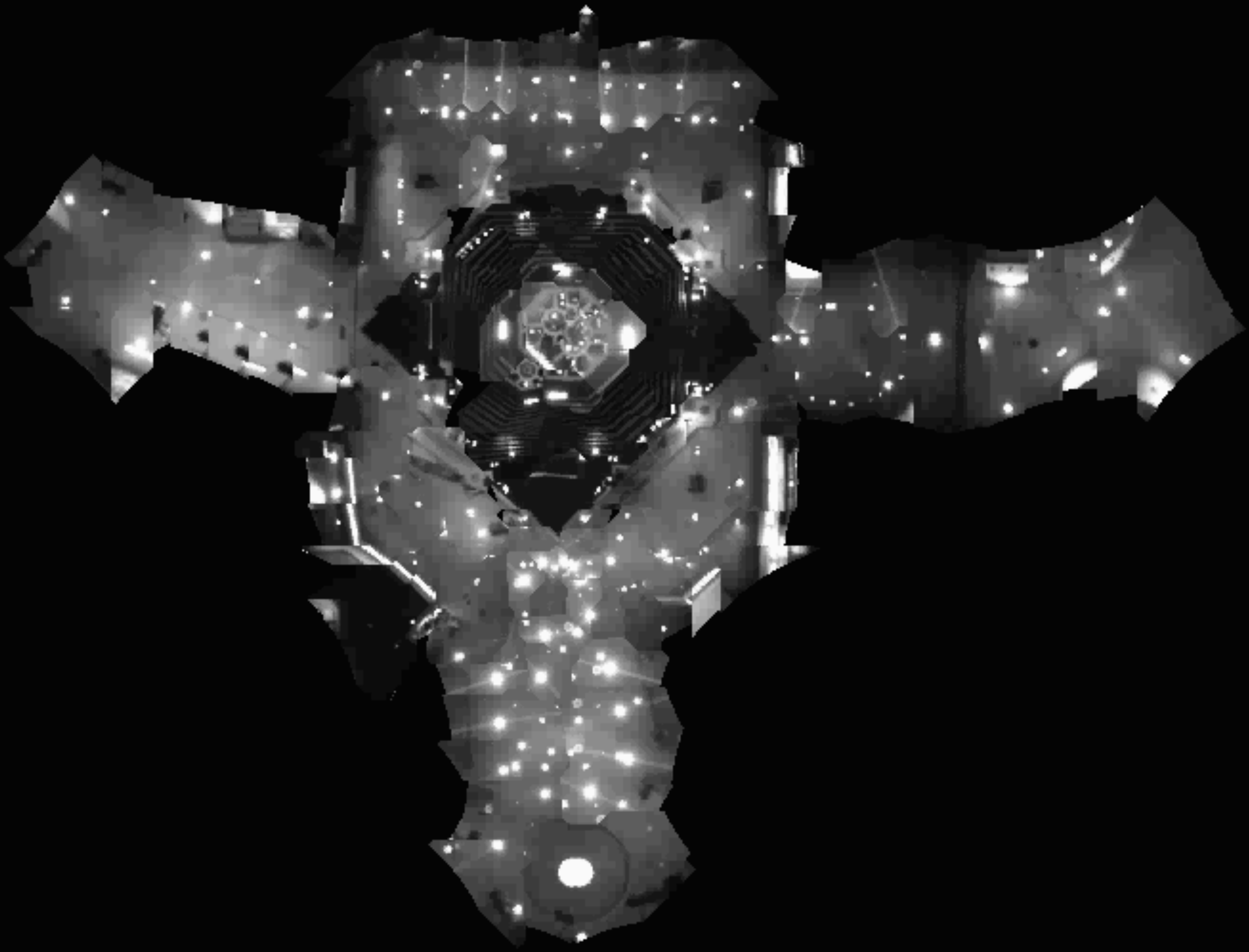

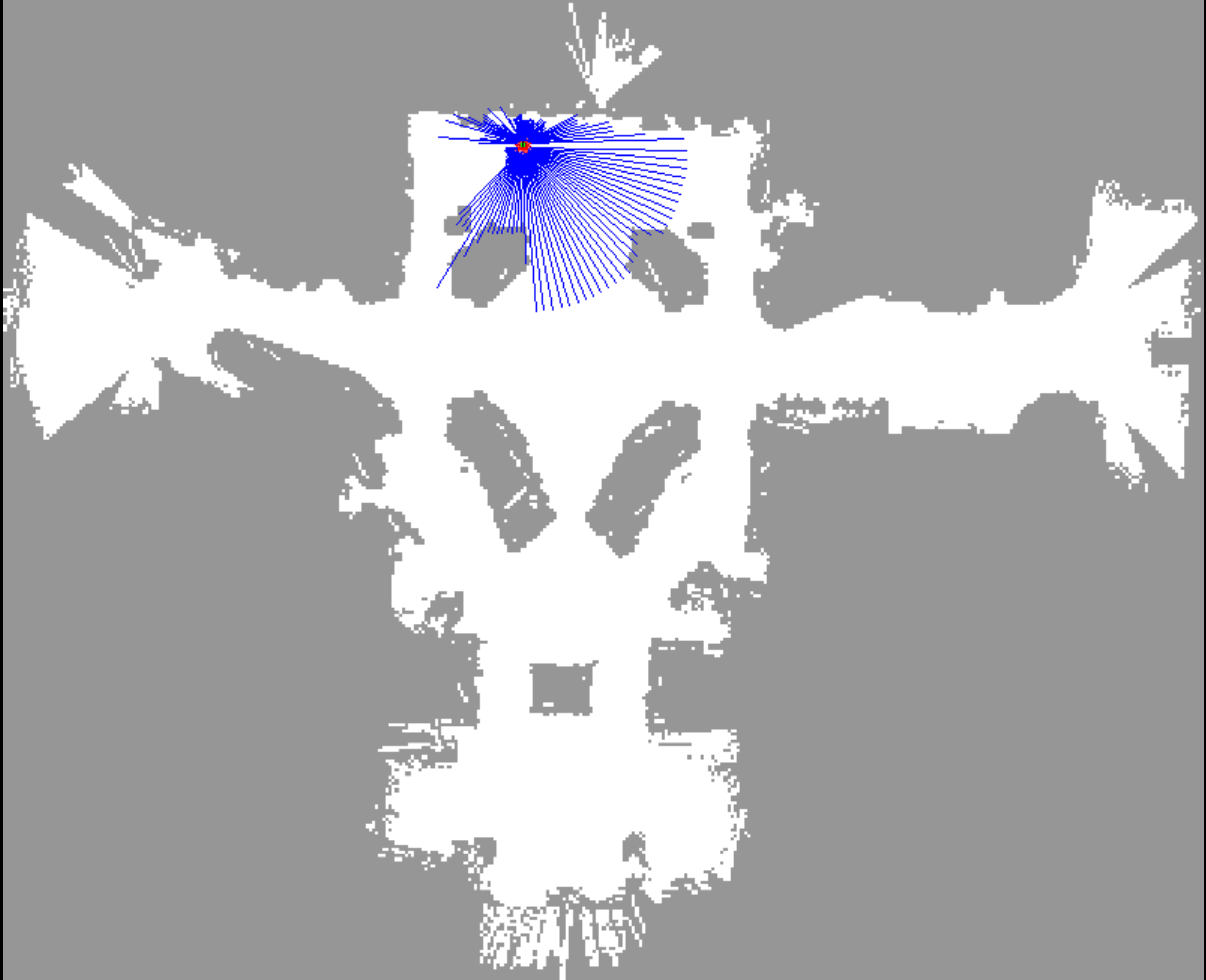

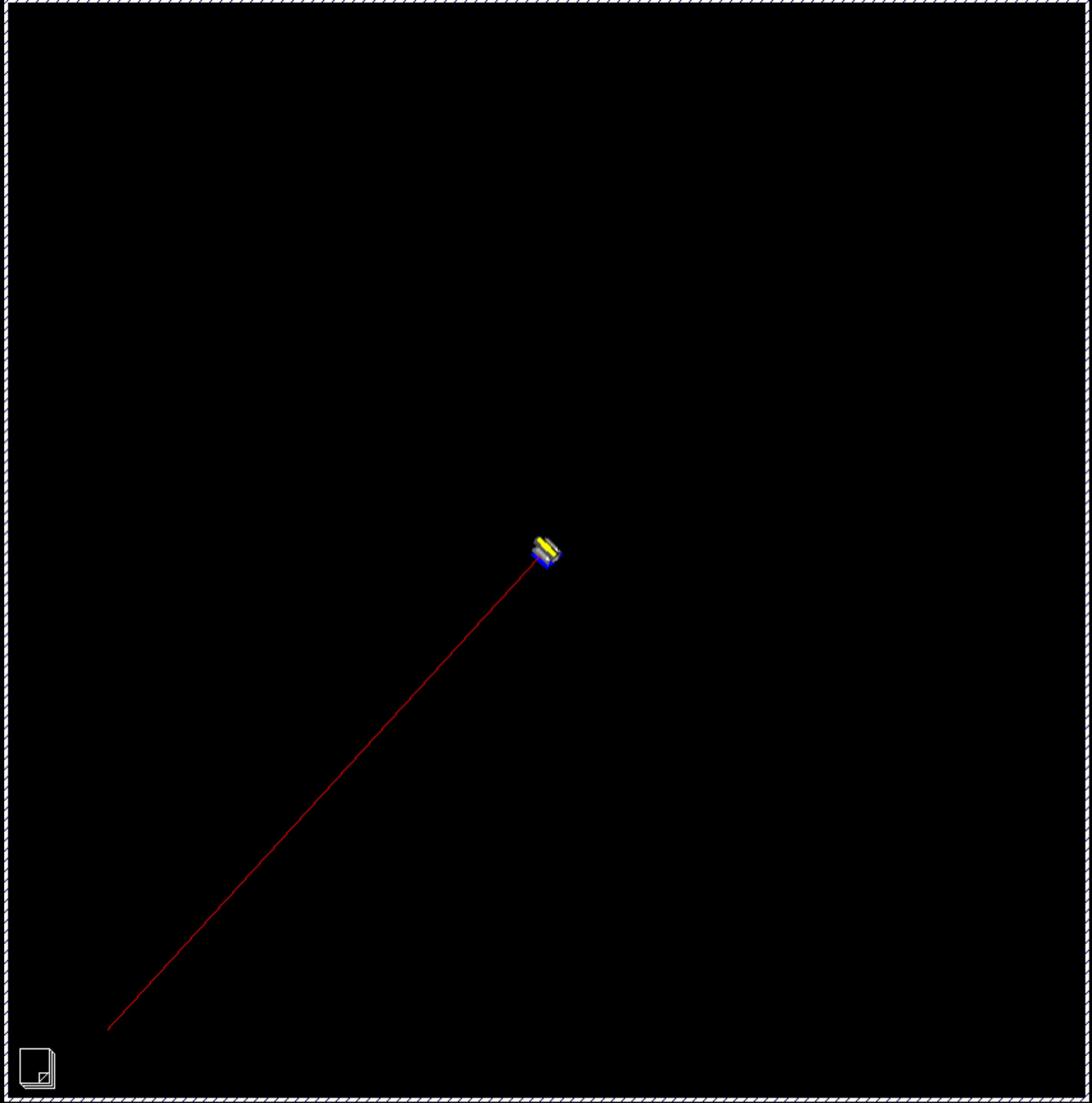

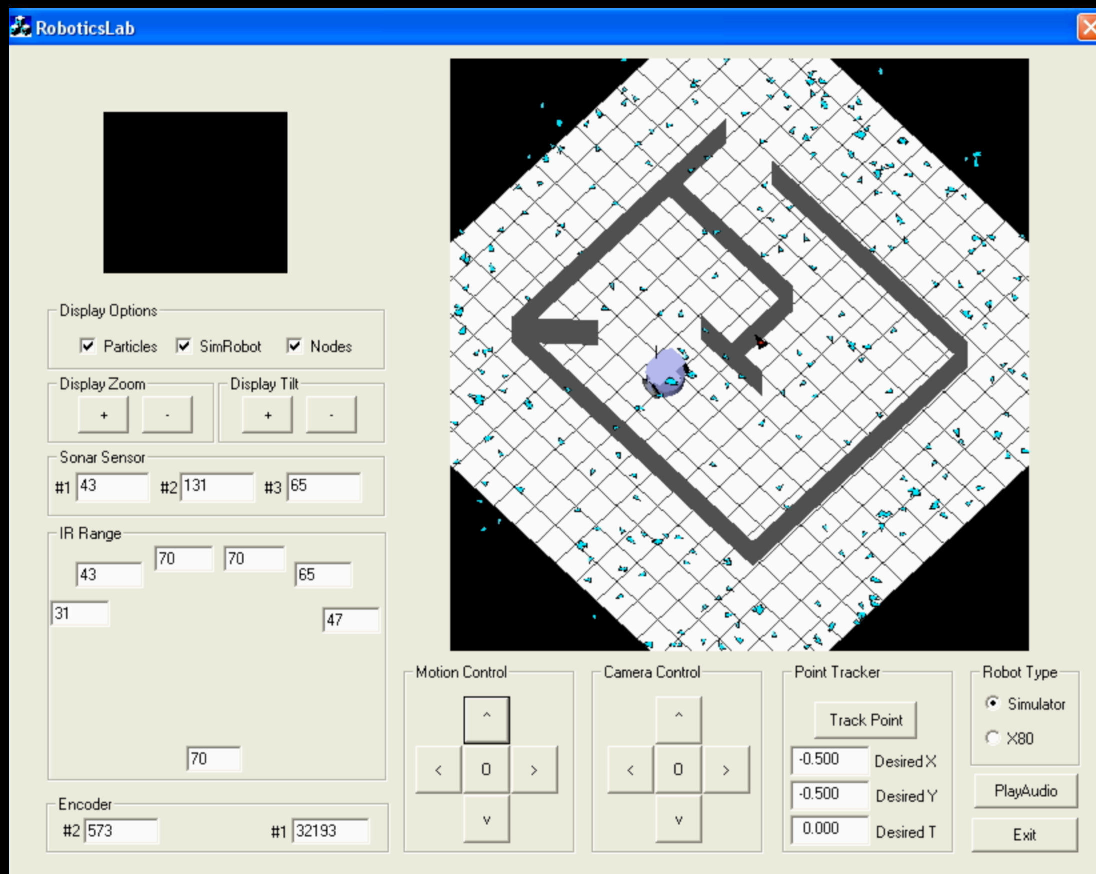
RoboticsLab
Display Options
Particles
SimRobot
Nodes
Display Zoom
+
-
Display Tilt
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Sonar Sensor
#1 43
#2 131
#3 65
IR Range
70
70
43
65
31
47
70
Encoder
#2 573
#1 32193
Motion Control
0
Camera Control
0
Point Tracker
Track Point
-0.500 Desired X
-0.500 Desired Y
0.000 Desired T
Robot Type
Simulator
X80
PlayAudio
Exit